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PROPAGATION OF THE WEAK-DISCONTINUITY SURFACE IN ORIENTED GLASS-REINFORCED PLASTICS WITH ALLOWANCE FOR THE RELAXATION OF THERMAL DISTURBANCES

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Within the framework of a generalized theory of heat conduction, the characteristic equation for glass-reinforced plastics has been obtained. Formulas for determination of the velocities of propagation of a thermal wave as functions of the slope of the normal to the characteristic surface and the angle of orientation of glass-fiber threads have been derived. The curves of inverse velocities and radial velocities have been constructed.

Composite materials formed by a combination of reinforcing elements (in the form of thin fibers or threads) and an isotropic binder are widely used in various fields of technology at present. The corrosion resistance and the electrotechnical and thermomechanical properties enable one to use glass-reinforced plastics under different conditions, including the case of exposure to laser radiation. Study of dynamic processes under such conditions requires that the finite velocity of propagation of heat be taken into account in connection with the very rapid character of heat release. Despite the fact that the time of relaxation of the heat flux is short ($\sim 10^{-11}$ sec), the velocity of propagation of thermal disturbances is a value of the order of 10^3 m/sec and can prove comparable to the velocity of movement of the heat source [1].

Let us consider an oriented single-layer glass-reinforced plastic in the plane $x_1 0 x_2$ (under plane deformation) produced by winding of the thread at an angle φ to the x_1 axis. Since the thermal and other mechanical properties of glass-reinforced plastics substantially depend on the orientation of glass-fiber threads (angle φ [2]), the corresponding system of equations of motion will be written in the following form (the internal heat sources are absent):

$$c_1(\varphi)\,\partial_1^2 u_1 + 2c_3(\varphi)\,\partial_1\partial_2 u_1 + c_5(\varphi)\,\partial_2^2 u_1 + c_3(\varphi)\,\partial_1^2 u_2 +$$

$$+ (c_2(\varphi)\, c_5(\varphi))\,\partial_1\partial_2 u_2 + c_4(\varphi)\,\partial_2^2 u_2 = \rho \ddot{u}_1 + \beta_{11}(\varphi)\,\partial_1 T + \beta_{12}(\varphi)\,\partial_2 T\,,$$

$$c_3(\varphi)\,\partial_1^2 u_1 + (c_2(\varphi) + c_5(\varphi))\,\partial_1\partial_2 u_1 + c_4(\varphi)\,\partial_2^2 u_1 + c_6(\varphi)\,\partial_2^2 u_2 +$$

$$+ 2c_4(\varphi)\,\partial_1\partial_2 u_2 + c_5(\varphi)\,\partial_1^2 u_2 = \rho \ddot{u}_2 + \beta_{21}(\varphi)\,\partial_1 T + \beta_{22}(\varphi)\,\partial_2 T\,, \tag{1}$$

$$\lambda_{11}(\varphi)\,\partial_1^2 T + 2\lambda_{12}(\varphi)\,\partial_1\partial_2 T + \lambda_{22}(\varphi)\,\partial_2^2 T - C\,(\dot{T} + \tau \ddot{T}) =$$

$$= T_0\,(\beta_{11}(\varphi)\,(\partial_1 \dot{u}_1 + \tau \partial_1 \ddot{u}_1) + \beta_{12}(\varphi)\,(\partial_1 \dot{u}_2 + \partial_2 \dot{u}_1 + \tau\,(\partial_1 \ddot{u}_2 + \partial_2 \ddot{u}_1)) + \beta_{22}(\varphi)\,(\partial_2 \dot{u}_2 + \tau \partial_2 \ddot{u}_2)\,,$$

$$\partial_i = \partial / \partial x_i\,, \quad i = 1, 2\,.$$

We prescribe the initial data for system (1) on the surface $Z(t, x_1, x_2) = 0$ and pass to the new variables $Z = Z(t, x_1, x_2)$ and $Z_{1,2} = Z_{1,2}(t, x_1, x_2)$ [3]. After the standard procedure we obtain the equation of weak discontinuity of system (1)

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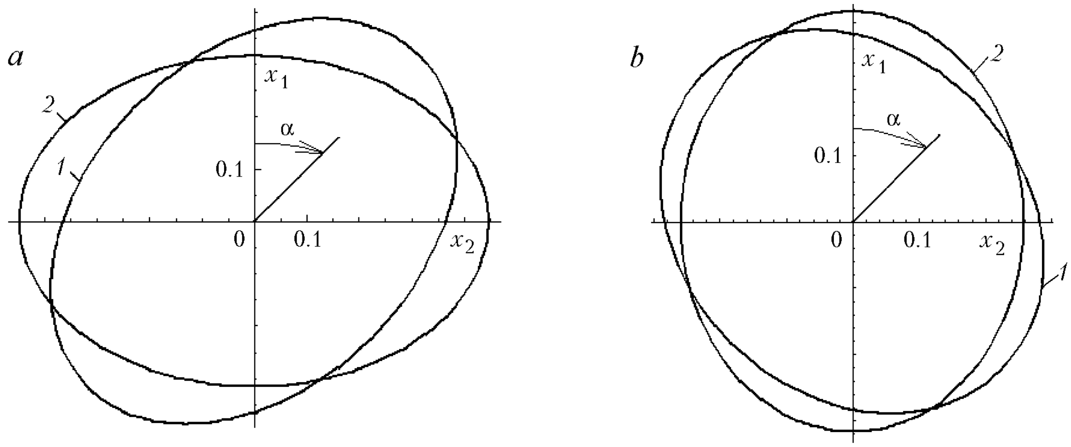


Fig. 1. Dimensionless curves of the ends of the vectors of refraction $r = 1/v$ of the thermal wave for $\lambda_{22}/\lambda_{11} = 0.5$ (a) and $\lambda_{22}/\lambda_{11} = 1.5$ (b): 1) $\varphi = \pi/4$; 2) $\pi/2$.

$$((c_1(\varphi)\, p_1^2 + 2c_3(\varphi)\, p_1 p_2 + c_5(\varphi)\, p_2^2 - \rho p_0^2)\,(c_3(\varphi)\, p_1^2 + (c_2(\varphi) + c_5(\varphi))\, p_1 p_2 +$$

$$+ c_4(\varphi)\, p_2^2) - (c_3(\varphi)\, p_1^2 + (c_2(\varphi) + c_5(\varphi))\, p_1 p_2 + c_4(\varphi)\, p_2^2)\,(c_6(\varphi)\, p_2^2 +$$

$$+ 2c_4(\varphi)\, p_1 p_2 + c_5(\varphi)\, p_1^2 - \rho p_0^2))\,(\lambda_{11}(\varphi)\, p_1^2 + 2\lambda_{12}(\varphi)\, p_1 p_2 + \lambda_{22}(\varphi)\, p_2^2 - C\tau p_0^2) = 0\,, \quad (2)$$

where $p_i = \partial Z/\partial x_i$ and $p_0 = \partial Z/\partial t$.

Equation (2) describes the propagation of quasitransverse and quasilongitudinal elastic waves and of a thermal wave. Further reasoning will be given only for the thermal wave, since data on the velocity of its propagation can be applied to calculation of the relaxation time of thermal disturbances analogously, for example, to [4].

For the velocity $V = -p_0/g$ of propagation of the thermal wave directed along the normal to the wave front, from (2) we obtain

$$V^2 = (\lambda_{11}(\varphi)\cos^2\alpha + \lambda_{12}(\varphi)\sin 2\alpha + \lambda_{22}(\varphi)\sin^2\alpha)/C\tau\,, \quad (3)$$

where $\cos\alpha = p_1/g$ is the direction cosine of the normal to the weak-discontinuity surface and $\sin\alpha = p_2/g$; $g^2 = p_1^2 + p_2^2$.

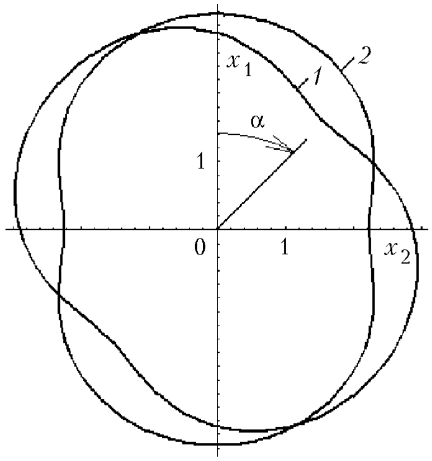
From (3) we find the dimensionless velocity of propagation of the thermal wave

$$v = \sqrt{(\lambda_{11}(\varphi)\cos^2\alpha + \lambda_{12}(\varphi)\sin 2\alpha + \lambda_{22}(\varphi)\sin^2\alpha)/\lambda_{11} n_*}\,.$$

The thermal-conductivity coefficients $\lambda_{ij}(\varphi)$ can be expressed in any revolved system by two basic thermal-conductivity constants — λ_{11} and λ_{22} — from the formulas [2]

$$\lambda_{11}(\varphi) = \lambda_{11}\cos^2\varphi + \lambda_{22}\sin^2\varphi\,, \quad \lambda_{22}(\varphi) = \lambda_{11}\sin^2\varphi + \lambda_{22}\cos^2\varphi\,, \quad \lambda_{12}(\varphi) = (\lambda_{22} - \lambda_{11})\cos\varphi\sin\varphi\,.$$

Figure 2 gives the dimensionless curves of the ends of the refraction vectors $r = 1/v$ for different angles of winding of the glass thread φ and ratios between the basic coefficients of thermal conductivity; the dimensionless parameter n_* is taken to be 0.1 (the characteristic frequency is of the order of 10^{11}–10^{12} 1/sec [5]). It is clear from the figure that the velocity of propagation of the thermal wave exceeds the velocity of propagation of the longitudinal elastic wave c_1 for both $\lambda_{22}/\lambda_{11} < 1$ and $\lambda_{22}/\lambda_{11} > 1$. Change in the angle of winding of the glass thread φ substantially influences the orientation of the r curve: for example, if φ changes by the angle $\Delta\varphi$ from zero to $\pi/2$ for $\lambda_{22}/\lambda_{11} < 1$, the r curve is rotated by the same, in practice, angle from the x_1 axis clockwise.



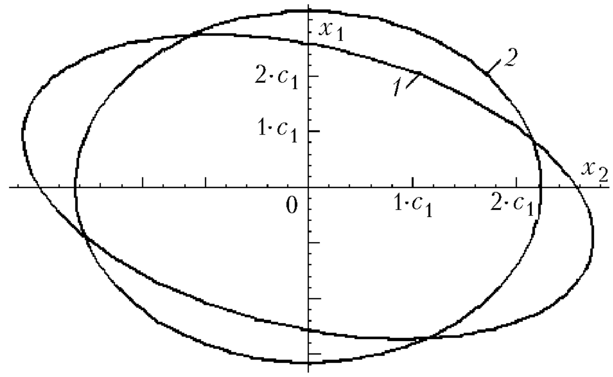


Fig. 2. Curves of dimensionless radial velocities $p = P/1$: 1) $\varphi = \pi/4$; 2) $\pi/2$.

Fig. 3. Thermal-wave fronts in the coordinate plane $x_3 = 0$: 1) $\varphi = \pi/4$; 2) $\pi/2$. The values plotted on the x_1 and x_2 axes are measured in meters.

Let us find the radial velocity of propagation of the thermal wave

$$P = \sqrt{\left(\frac{\partial p_0}{\partial p_1}\right)^2 + \left(\frac{\partial p_0}{\partial p_2}\right)^2} \,. \tag{4}$$

We express p_0 from Eq. (3) and find partial derivatives with respect to p_1 and p_2. Taking into account that $p_1 = g \cos \alpha$ and $p_2 = g \sin \alpha$, we have

$$\frac{1}{c_1}\frac{\partial p_0}{\partial p_1} = \frac{\lambda_{11}(\varphi)\cos\alpha + \lambda_{12}(\varphi)\sin\alpha}{\sqrt{n_*\left(\lambda_{11}(\varphi)\cos^2\alpha + \lambda_{12}(\varphi)\sin 2\alpha + \lambda_{22}(\varphi)\sin^2\alpha\right)}} \,,$$
$$\frac{1}{c_1}\frac{\partial p_0}{\partial p_2} = \frac{\lambda_{12}(\varphi)\cos\alpha + \lambda_{22}(\varphi)\sin\alpha}{\sqrt{n_*\left(\lambda_{11}(\varphi)\cos^2\alpha + \lambda_{12}(\varphi)\sin 2\alpha + \lambda_{22}(\varphi)\sin^2\alpha\right)}} \,. \tag{5}$$

Having substituted expressions (5) into (4), we obtain the dimensionless radial velocity $p = P/c_1$ of propagation of the thermal wave as a function of the slope of the normal of the characteristic surface. Figure 2 shows the curves of dimensionless radial velocities for different angles of winding of the glass thread φ and for $\lambda_{22}/\lambda_{11} = 0.5$, and $n_* = 0.1$.

The dependence of the dimensionless radial velocity p on the slope α of the normal to the characteristic surface is more pronounced than the dependence of v on α (see Fig. 1a and Fig. 2). A comparative analysis of the values of the velocities P and V for the same values of the angle of winding of the glass thread shows that the radial velocity only slightly exceeds the velocity of propagation of the thermal wave along the normal to the characteristic surface. Thus, the largest deviation of the velocities, for example, when $\lambda_{22}/\lambda_{11} = 0.5$, is observed for angles α multiple to $\pi/4$ and amounts to $\approx$6%.

We note that the curves of dimensionless radial velocities presented in Fig. 2 enable one to determine the radial velocity as a function of the slope of the normal to the characteristic surface but they are not the wave fronts of the thermal wave in the coordinate plane $x_3 = 0$. To construct the thermal-wave front we find the expressions for the coordinates (x_1, x_2) of points of the medium which have been approached by the wave-disturbance energy by the instant t. We take into account the following equalities [3]:

$$\frac{dx_i}{dt} = \frac{\partial p_0}{\partial p_i} \, . \tag{6}$$

The right-hand side of (6) is independent of time; therefore, integrating (6) under the assumption that the disturbance source is in the origin of coordinates, we obtain

$$x_i = \frac{\partial p_0}{\partial p_i} \, t \, . \tag{7}$$

Expressions (7) enable us to determine the coordinates of a point of the thermal-wave front as quantities which are in proportion to the velocity of propagation of the longitudinal elastic waves, i.e., to find the absolute values of the coordinates x_1 and x_2 at the instant t we must multiply them by the velocity c_1. Figure 3 shows the thermal-wave fronts for different angles of winding of the glass thread φ ($\lambda_{22}/\lambda_{11}$ = 0.5 and n_* = 0.1).

In closing, we note that in a more accurate formulation of the problem on wave motions in an oriented glass-reinforced plastic, one must take into account different values of the relaxation times of thermal disturbances, which correspond to the main directions of heat conduction of the glass-reinforced plastic.

NOTATION

C, specific heat at constant deformation; c_1, velocity of propagation of the longitudinal elastic wave; $c_k(\varphi)$, elastic constants; $n_* = \tau\omega_*$, characteristic number of vibrations over the period of relaxation of the heat flux; T, absolute temperature; $\mathbf{u} = (u_1, u_2)$, displacement vector; $v = V/c_1$, dimensionless velocity of propagation of the discontinuity surface; α, slope of the normal to the weak-discontinuity surface; $\beta_{ij}(\varphi)$, thermomechanical constants; φ, angle of winding of the glass thread; λ_{11} and λ_{22}, basic thermal-conductivity constants; $\lambda_{ij}(\varphi)$, thermal-conductivity coefficients; τ, relaxation time of thermal disturbances; $\omega_* = c_1^2 C/\lambda_{11}$, characteristic quantity having the dimension of frequency. Subscripts: i, j = 1 and 2; $k = \overline{1, 5}$; point, differentiation with respect to time.

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